# SOME RECENT THEORETICAL STUDIES ON OPEN MICROSTRIP

Edward F. Kuester and David C. Chang
Electromagnetics Laboratory
Department of Electrical Engineering
University of Colorado
Boulder, Colorado 80309

ABSTRACT

An approximate analytical theory for open microstrip recently developed by the authors was compared with existing theoretical results; however detailed examination pointed up discrepancies among these results for strips of width comparable to or greater than substrate thickness. Since these disagreements appear to arise from approximations made on the charge and current distributions on the strip, the authors undertook an investigation of these distributions for wide microstrip which are reported here. Significant differences are found in contrast to the case of narrow strips, which can affect the accuracy of numerical procedures for finding the effective dielectric constant. An analytical approach to excitation of microstrip by an idealized voltage generator is also discussed.

## Introduction

In a recent analytical study of narrow open microstrip (Fig. 1) a literature search was made to find numerical results with which the new, approximate theory could be compared.[1] Somewhat surprisingly, comparison of the half dozen or so existing theoretical results showed discrepancies in effective dielectric constant of up to 25% for strips whose width is comparable with substrate thickness.[2] The main reason for these discrepancies seems to be an inadequate knowledge of current and charge distributions on the strip--many moment-method techniques, for instance, use piecewise-constant approximating functions and encounter difficulties with spurious solutions for the effective dielectric constant, $\varepsilon_{r_{eff}}$.

Moreover, it turns out that the forms of the charge and current distributions are different from each other for wider strips (the charge distribution is slightly "flatter" in that it decays more rapidly away from the edges of the strip). A study was therefore undertaken to find, if possible, simple but accurate forms for these distributions on wider microstrips, and to use these to find accurate values for $\varepsilon_{r_{eff}}$.

In the latter part of this paper we address the problem of how to find the excitation of a microstrip by an idealized voltage source. This problem requires finding not only the excitation coefficient of the transmission mode, but also the accompanying radiation into both the space above the strip and the surface wave along the substrate. A full modal description for the current is developed, which should also be useful in attacking microstrip discontinuity problems, such as reflection and radiation at a junction or at an open-ended microstrip.

## Change and Current Distributions on the Strip

We can start with the integral equations for charge and current derived earlier[1] by the authors. We have

$$\int_{-\ell}^{\ell} G_e(y-y')\rho_1(y')dy' = \cosh\sqrt{\alpha^2-1}\, k_o y; \qquad |y| \le \ell \qquad (1)$$

where $\rho_1(y)$ is the (normalized) charge distribution on the strip ($-\ell \le y \le \ell$), and

$$G_e(y) = 2\int_0^{\infty} \frac{(u_n \tanh u_n T)\cos k_o\lambda y}{\varepsilon_r u_o + u_n \tanh u_n T}\,\frac{d\lambda}{u_o} \qquad (2)$$

$T = k_o t$ is the substrate thickness normalized to free-space wavenumber, while

$$u_n = (\lambda^2+\alpha^2-\mu_r\varepsilon_r)^{\frac{1}{2}}\ ; \quad u_o = (\lambda^2+\alpha^2-1)^{\frac{1}{2}};\ \mathrm{Re}(u_o) \ge 0 \qquad (3)$$

and $\mu_r$ and $\varepsilon_r$ are respectively the relative permeability and permittivity of the substrate. A propagation factor of $\exp(i\omega t - ik_o\alpha x)$ has been assumed, where $\alpha^2 = \varepsilon_{r_{eff}}$. Once the solution of (1) is known as a function of $\alpha$, the longitudinal current density $J_x(y)$ is found from

$$\int_{-\ell}^{\ell} G_m(y-y')J_x(y'')dy' = \frac{i\alpha}{k_o}\Big[\cosh\sqrt{\alpha^2-1}\, k_o y + \int_{-\ell}^{\ell} M(y-y')\rho_1(y')dy'\Big] \qquad |y| \le \ell \qquad (4)$$

where

$$G_m(y) = 2\mu_r \int_0^{\infty} \frac{\cos k_o\lambda y\, d\lambda}{\varepsilon_r u_o + u_n \coth u_n T} \qquad (5)$$

$$M(y) = 2(\mu_r\varepsilon_r - 1)\int_0^{\infty} \frac{\cos k_o\lambda y \qquad d\lambda}{(\varepsilon_r u_o + u_n \tanh u_n T)(\mu_r u_o + u_n \coth u_n T)u_o} \qquad (6)$$

Knowing both current and charge as functions of $\alpha$, we then enforce an edge condition on the transverse current density on the strip to obtain a characteristic equation for determining $\alpha$:

$$\int_{-\ell}^{\ell} [ik_o\alpha\, J_x(y) + \rho_1(y)]dy = 0 \qquad (7)$$

Although alternative forms of these integral equations exist, these are most convenient for our purposes since the unknown functions are the important physical quantities of charge and longitudinal current, which are closely related to concepts of capacitance and inductance of the line, and in addition possess the same type of singular behavior near the edges of the strip.

Under the assumptions that $\varepsilon_r \gg 1$ and $\mu_r = 1$ (quite typical for microstrip), and that no dimension is yet comparable to a free-space wavelength, it is found that the kernels (2) and (5) can be approximated accurately by

$$G_e(y) \simeq -\frac{2}{\varepsilon_r+1}\,\ln\Big[\tanh\frac{\pi|y|}{4t}\Big] \qquad (8)$$

$$G_m(y) \simeq \ln\frac{\sqrt{y^2+4t^2}}{|y|} \qquad (9)$$

Now (8) is simply the kernel for the integral equation determining the charge distribution on a <u>symmetric</u> (semi-closed) stripline[3] (i.e., a stripline with an additional ground plane located a distance t <u>above</u> the strip - see Fig. 2(a)). The approximate equivalence of symmetric and open striplines for large values of $\varepsilon_r$ has been observed previously.[4,5] Thus, we can say with confidence that the charge distribution can be represented accurately by a function of the form:

$$\rho_1(y) \simeq \frac{\rho_o}{\sqrt{\cosh^2(\frac{\pi\ell}{2t}) - \cosh^2(\frac{\pi y}{2t})}}; \quad |y| \le \ell \qquad (10)$$

where $\rho_0$ is a constant.

The kernel (9), on the other hand, is the kernel for the integral equation determining the current distribution on the parallel-plate line in free-space shown in Fig. 2(b). While this distribution is known exactly, it is a quite complicated expression involving elliptic integrals and requires solution of an implicit transcendental equation. The authors have obtained a closed-form approximation for this distribution which is quite accurate for any $\ell/t$ ratio; it has the proper limiting behavior as $\ell/t \to \infty$ and as $\ell/t \to 0$, and at worst (near $\ell/t = 2$) is in error by about 12%, and then only in the region close to the edge of the strip (Fig. 3). This approximation is similar to (11):

$$J_x(y) \approx \frac{J_0}{\sqrt{\cosh^2(\frac{\pi\ell}{4t}) - \cosh^2(\frac{\pi y}{4t})}} \qquad (11)$$

By comparison with (10) it can be seen that the charge distribution is different from that of the current; it is "flatter," i.e., decays more rapidly away from the singularities at the edges of the strip.

The distributions (10) and (11) can be used, for example, with the variational expression given by Pregla and Kowalski[6] to obtain a dispersion equation for $\alpha$. Their formulation requires the knowledge of the Fourier transforms of (10) and (11) with respect to y; fortunately these are known exactly[7] as the Legendre functions $P_{-\frac{1}{2}+i2\tau}(\cosh \frac{\pi\ell}{2t})$ and $P_{-\frac{1}{2}+i2\tau}(\cosh\frac{\pi\ell}{2t})$ respectively, where $\tau = \lambda T/\pi$, and $\lambda$ is the transform variable, similar to that used in (2), (5) and (6). Efficient means for computing these functions numerically exist[7-9] for all ranges of $\lambda$ and $\ell/t$.

The resulting dispersion relation is similar to that found earlier for a narrow strip:[1]

$$\alpha^2 = L(\alpha)C(\alpha) \qquad (12)$$

where $L(\alpha)$ and $C(\alpha)$ consist of a static part and a dispersive part,

$$L(\alpha) = L_s + L_d(\alpha) \qquad (13)$$

$$\frac{1}{C(\alpha)} = \frac{1}{C_s} + \frac{1}{C_d(\alpha)} \qquad (14)$$

and $L_d(\alpha)$ and $C_d(\alpha)$ are given by Sommerfeld integrals similar to (2), (5) and (6), involving the Legendre functions referred to above as well. Agreement with the results of Jansen[10] is shown in Fig. 4, but since only a single function is needed to represent each of the charge and current distributions, computation time is reduced over the moment method used by Jansen which requires a large number of basis functions for wide strips. Comparison with results for even wider strips obtained by Wiener-Hopf methods[11] is also quite good in view of the fact that such methods do not have their greatest accuracy at low frequencies.

## Excitation by a Slot Voltage Generator

To consider excitation of the strip by a delta-function slot voltage generator, we can write the fields, charges and currents as Fourier transforms in to recover the longitudinal dependence, e.g.,

$$e_x(x) = \int_{-\infty}^{\infty} E_x(\alpha)e^{-ik_0 x} d\alpha \qquad (15)$$

The boundary-source condition $e_x(x) = -V_0\,\delta(x)$ on the strip leads, instead of (1), to the integral equation

$$\int_{-\ell}^{\ell} G_e(y-y')\rho_1(y')dy' = \frac{k_0\alpha}{2\eta_0(\alpha^2-1)} V_0 + A\cosh\sqrt{\alpha^2-1}\,k_0 y \qquad (16)$$

where $\eta_0 = \sqrt{\mu_0/\varepsilon_0}$ and A is a constant to be determined. In special cases such as narrow strip,[1] or the dense substrate ($\varepsilon_r >> 1$) considered above, good approximate expressions for $\rho_1(y)$ are available. For the narrow strip, for example, the remainder of the analysis proceeds as for the sourceless case,[1] and leads to the following expression for the current density on the strip:

$$j_x(x,y) = (\text{const}) \frac{V_0}{(\ell^2-y^2)^{\frac{1}{2}}} \int_{-\infty}^{\infty} \frac{e^{-ik_0\alpha x}}{M(\alpha)} d\alpha \qquad (17)$$

where

$$M(\alpha) = \frac{\alpha^2}{L(\alpha)} - C(\alpha) \qquad (18)$$

A modal decomposition of this current follows if we deform the integration contour in (17) upwards (if x < 0) or downwards (if x > 0) over the singularities of $M(\alpha)$ in the appropriate half of the $\alpha$-plane. These include[1] a pair of branch cuts at $\alpha = \pm 1$, corresponding to waves radiated into the space above the strip; another pair of cuts at $\alpha = \pm\alpha_p$ (the propagation constant of the $TM_0$ surface wave supported by the substrate), corresponding to waves radiated from the strip along the substrate as a surface wave, and the pole located at the zero of $M(\alpha)$ (the root of (12)). The branch cuts are illustrated in Fig. 5. Numerical evaluation of the branch cut integrals will show the individual contributions of surface wave and "space wave" to the radiation of this structure, and it seems likely that microstrip of finite length might be analyzed by suitable adaptations of these results, in a manner similar to that which is used in the study of finite cylindrical antennas.

## References


1. D.C. Chang and E.F. Kuester, to appear in Arch. Elek. Übertragungstech.
2. E.F. Kuester and D.C. Chang, to appear in IEEE Trans. MTT.
3. G. Kowalski and R. Pregla, Arch. Elek. Übergragungstech. v. 27, pp. 51-52 (1973).
4. J.M.C. Dukes, Proc. IEE (London) part B, v. 103, pp. 319-333 (1956).
5. H. A. Wheeler, IEEE Trans. MTT v. 13, pp. 172-185 (1965).
6. R. Pregla and G. Kowalski, Arch. Elek. Ubertragungstech, v. 28, pp. 339-340 (1974).
7. M.I. Zhurina and L.N. Karmazina, Tables and Formulae for the Spherical Functions $P^m_{-\frac{1}{2}+i\tau}(Z)$. Oxford: Pergamon Press, 1966.
8. F.W.J. Olver, Asymptotics and Special Functions. New York: Academic Press, 1974.
9. W. Bartky, Rev. Mod. Phys. v. 10, pp. 264-269 (1978).
10. R.H. Jansen, IEEE Trans. MTT v. 26, pp. 75-82 (1978).
11. E.I.Nefedov and A.T. Fialkovskii, Sov. Phys. Doklady v. 22, pp. 448-450 (1977).

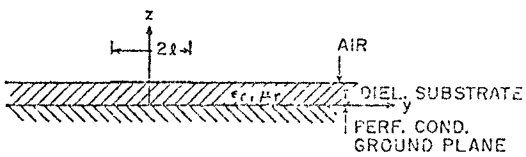


Fig. 1 Open microstrip

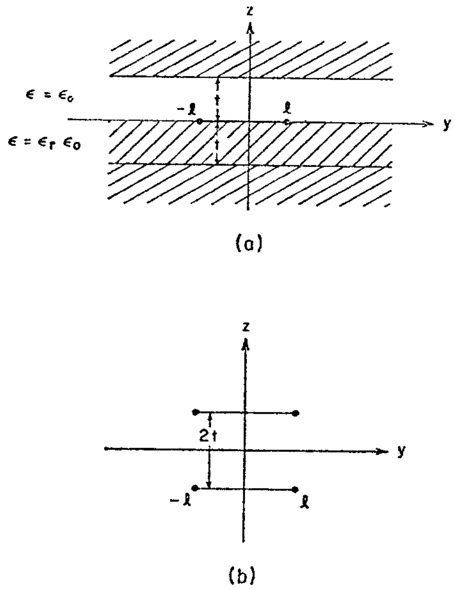


Fig. 2 (a) Symmetric stripline
(b) Parallel-plate line

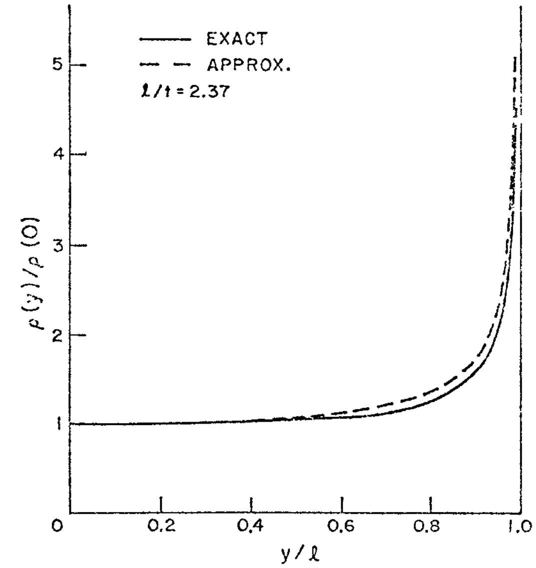


Fig. 3 Comparison of exact and approximate charge distributions for parallel strips

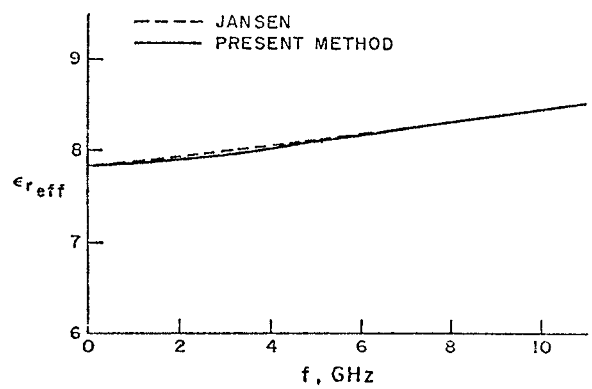


Fig.4 Effective dielectric constant $\varepsilon_{r_{eff}} \equiv \alpha^2$ for open microstrip; $t = 0.64$ mm, $\ell = 1.5$ mm, $\varepsilon_r = 9.9$ as computed by Jansen, and by the present method

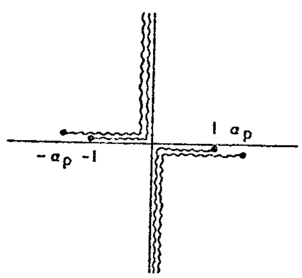

Fig. 5 Branch cuts in $\alpha$-plane